Quantitative solution methods to the management of human resources in a multi-project environment

Thomas Felberbauer [a], Karl F. Doerner [b]

[a] FH OÖ Forschungs & Entwicklungs GmbH - Department of Operations Management, Wehrgrabengasse 1-3, A-4400 Steyr, AUSTRIA

[b] University Vienna – Department of Business Administration, Oskar-Morgenstern-Platz 1, A-1090 Vienna, AUSTRIA

KURZFASSUNG/ABSTRACT:

For the problem of simultaneous scheduling projects and assigning multi-skilled resources two practical model extensions, interruption between work packages and the integration of labor contracts are presented. The objective of the mixed-integer linear program is to minimize external and internal labor costs. For the generated test instances the Key Performance Indicators (KPIs) solvability, solution time and solution gap of the MIP-solver are investigated. Due to the fact that the exact solver fails to find feasible solutions for complex instances, a solution method is provided which decomposes the problem into a scheduling and a staffing sub problem. The solution approach is a hybrid metaheuristic by means of iterated local search. The conducted study shows that the developed solution method performs favourably for small and medium-sized test instances and additionally finds good solutions for instances where the exact solver fails.

Keywords: Project Scheduling, Staffing, Resource Assignment, Labor Contracts, Mixed Integer Programming, Hybrid Metaheuristics

1 INTRODUCTION

Expending resources efficiently, especially in high-wage countries and labor-intensive industries, is critical to global competitiveness. Yet the related, inevitable resource planning tasks pose a huge challenge to companies that manage vast resources. In basic hierarchical decision models for production or personnel planning (e.g., Hax and Meal (1975), Mundschenk and Drexl (2007)) project scheduling and personnel planning usually involve long-term decisions. Because these models feature aggregated data, they use deterministic optimization techniques such as linear programming (LP) or mixed integer programming (MIP) standard solvers (e.g. CPLEX) to solve the aggregate planning problem.

The multi-mode resource constrained project scheduling problem (MRCPSP) deals with the problem of assigning multi skilled human resources to work, taking into account resource-specific skills and efficiencies. The objective of the model is to minimize the capacity costs of internal and external resources with respect of the demand fulfillment.

For a survey of exact and heuristic solution methods and extensions of the resource constrained project scheduling problem (RCPSP), we refer to (Kolisch and Padman, 2001) and (Hartmann and Briskorn, 2010). A state-of-the-art literature review on personnel scheduling, analyzing 293 articles from 2004 to 2012, is provided by (van den Bergh et al., 2013). They used a classification of personnel members according to their labor contract, such that they distinguished between full-time and part-time workers. The authors note that the vast majority research focuses on full-time personnel problems, which highlights a contribution of our paper. A solution approach using variable neighborhood search, presented by (Fleszar and Hindi, 2004), focuses on the non-preemptive and non-renewable RCPSP problem with the goal of minimizing the total make span.

Substantial literature focuses on project scheduling, with and without the possibility of preemption or interruption, personnel planning, considering different kinds of labor contracts, staffing, and heterogeneous skills. However, to the best of our knowledge, no previous studies have combined these aspects and provided an appropriate solution method.

In this study, we consider the problem of long-term personnel planning and the simultaneous project scheduling by minimizing internal and external labor costs. With this approach, we extend the tight MIP-formulation by (Heimerl and Kolisch, 2010) by allowing the interruption between work packages and introducing decisions about different labor contracts, namely full-time, part-time, or no employment.

For the developed model the performance of an exact and a hybrid metaheuristic solution method is investigated for small to large test instances.

2 HEURISTIC SOLUTION METHOD

Figure 1 shows the framework of the developed hybrid metaheuristic. The main components are the iterated local search metaheuristic (ILS) (Lourenço, Martin and Stützle, 2010) and the exact post-processing.

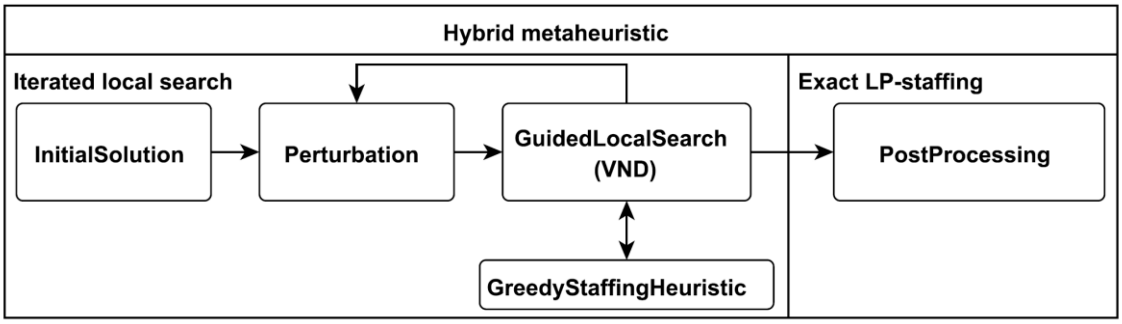

Figure 1. Hybrid metaheuristic framework

After generating an initial solution we use a variable neighborhood descent (VND) (Hansen et al., 2010) as guided local search (GLS) algorithm. The neighborhood operators use information about the capacity profiles that guide the VND search to prioritize time periods with high external or overtime costs. For the evaluation of the predefined project work package starting times, according to the neighborhood function, we use a greedy staffing heuristic. The perturbation operator is applied if all neighborhoods cannot improve the incumbent solution. It generates a new solution randomly and helps to escape from a local optimum. The ILS approach with the implemented VND is repeated until a stopping criterion is met.

Finally, in a post-processing step, the staffing decisions' of the best schedules obtained by the ILS are reevaluated with the exact LP solver. Therefore, the top ILS project schedules become the input for the linear staffing problem, which is optimized using CPLEX.

3 COMPUTATIONAL STUDIES

For the computational studies test instances are generated using the same test-instance-generator as (Heimerl and Kolisch, 2010). Contrary to their work, in this paper, bigger and therefore more complex problems are solved using the supercomputer "Mach" of the Johannes Kepler University of Linz.

In the first computational study, the influence of the model extension *MIP-SP* on the KPIs (e.g.: solution time, solution gap and solvability) are compared to the *basic* model formulation using the exact solver. It is investigated how the model extension influences the KPIs and how long the exact solver is an appropriate method to solve the problems. Due to the fact that the first study shows that the exact solver has problems in solving large and complex problems, even with unrealistic long solver-time-limits, the need of an appropriate heuristic solution method is illustrated.

4 RESULTS

In the first numerical study, we investigate the model complexity of the project scheduling and personal planning models *MIP-II* and *MIP-SP* to determine when the exact solver is an appropriate method to solve the models and at which point on a heuristic solution method is necessary.

Table 1 shows the solution time, the solution gap, and the solvability for the two different model formulations and three different time limits of the MIP solver (360 seconds, 1 hour, and 10 hours). For the study with a time limit of 360 seconds per instance, all 195 test instances are solved once; for the study with a time budget of 1 hour per instance, one combination of the master data, amount of projects $|P|$ and the time window size γ, or 39 instances, are solved. Finally, for the numerical study with a time limit of 10 hours per instance only the test instances with a time window size $\gamma = 3$ from the subset of 39 instances are solved, due to the limited experimental time budgets. All test instances are solved using the state-of-the-art solver CPLEX, parametrized with default values. The solution time (st) is the average solution time in seconds for all feasible solved test instances. The average solution gap (sg), measured as a percentage, is listed in the next column of **Table 1**. The key performance indicator solvability (sa) reveals the percentage of feasible solved test instances. A feasible solved test instance is a test instance in which the solver can find a feasible integer solution within the specified time limit.

Table 1. Solution time (st), solution gap (sg), and solvability (sa) for the problem extension, CPLEX time limits, and time window size γ

problem abbr.	Cplex(360sec.) $\gamma \in \{1,2,3\}$			Cplex(1h.) $\gamma \in \{1,2,3\}$			Cplex(10h.) $\gamma = 3$		
	st [sec.]	sg [%]	sa [%]	st [sec.]	sg [%]	sa [%]	st [sec.]	sg [%]	sa [%]
MIP-II	292	0.08	100	2,064	0.03	100	28,337	0.03	100
MIP-SP	353	0.83	66	3,482	1.5	89.7	36,000	3.45	100

The average solution time and solution gap for all test instances solved by CPLEX parametrized with default values and a time limit of 360 sec. are 292 sec. and 0.08% solving the *MIP-II* model. In addition *MIP-II* can find a feasible solution for all test instances. A tenfold increase of the time budget from 360 seconds to 1 hour increases the solvability reduces the average solution gap from 0.08% to 0.03%. Another tenfold increase of the time limit to 10 hours per instance results in an average solution time of 28,337 seconds and a solution gap of 0.03% for the *MIP-II* model formulations. Investigating only those instances for which the time window size $\gamma = 3$, we find that a hundredfold increase of solution time leads to an average improvement of the solution gap from 0.16% to 0.03%.

For the *MIP-SP* model, CPLEX, with default parameterization and a time limit of 360 seconds, finds a solution for 66% of all test instances within an average solution time of 353 sec. and an average solution gap of 0.83%. For project amounts of $|P| \geq 200$ and $\gamma = 1$, $|P| \geq 130$ and $\gamma = 2$, and $|P| \geq 50$ and $\gamma = 3$, CPLEX fails to find feasible solutions. In this cases the exact solver is not a reliable solution method. With an increase of the CPLEX time limit to 1 hour, the solvability percentage rises to 89.7%, the solution time to 3,482 seconds, and the solution gap to 1.5%. A time budget of 10 hours and test instances with $\gamma = 3$ results in average solution time of 36,000 seconds, and an average solution gap of 3.45%

For the practical, developed model extensions *MIP-SP*, the exact solver has problems solving medium and large test instances, even with unrealistically long time limits. Especially for medium and large sizes, the MIP solver is not a reliable solution method and practically not usable. This study confirms the need for an appropriate heuristic solution method for the *MIP-SP* model formulation.

The results of the developed hybrid metaheuristic solution method for the *MIP-SP* problem, considering the decisions on labor contracts, project scheduling with interruption, and personnel staffing are presented in **Table 2**. A summary of the average solution *sg'* with respect to the amount of projects $|P|$ and the time window size γ is given. Each of the five generated instances with the same combination of $|P|$ and γ are solved with the heuristic, using 10 replications to account for the stochastic value of the developed metaheuristic. For the heuristic solution method, 360 seconds represents the stopping criterion, and we conducted the *PostProcessing* afterwards. For a fair comparison of the developed heuristic and the MIP solver, the average solution time of the heuristic solution method (including the required post-processing time) becomes the new time limit for a new experiment, solving all test instances with the exact solver. Therefore, the results regarding solvability differ slightly from the results reported in **Table 1**. To tackle the potential problem if the MIP solver cannot find any feasible integer solution within the time budget together with the need for a key performance indicator that measures the performance of the heuristic for big and complex instances, we calculate the solution gap (*sg'*) with respect to the lower bound per instance. This boundary is determined by relaxing all binary and integer decision variables and solving the pure linear model. To show the variance of the results, we report the 95% confidence interval ±*CI* of the solution gap *sg'*. For the heuristic solution gap, the variance is twofold. First, variance arises from the five different test instances with one combination of $|P|$ and γ. Second, variance emerges from the ten replications used to solve one test instance. For the calculation of confidence interval ±*CI* using the exact solver, we do not use replications, so the variance is caused only by the five test instances with the same problem characteristic of $|P|$ and γ. The solution gaps of the MIP solver results indicated with an asterisk (*) are combinations of $|P|$ and γ where CPLEX cannot find a feasible integer solution for all five instance combinations within the predefined time limit. If only one problem characteristic could be solved, the confidence interval is empty. In the bottom row, the average values over all amount of projects $|P|$ are reported per time window size γ. The bold reported solution gaps indicate instance combinations, where the heuristic outperforms the exact solver, either in the sense of solvability or in the sense of a solution gap.

Table 2. MIP-SP: ILS vs. CPLEX(st-ILS) solution gaps (sg') compared with the lower bound and its 95% confidence interval (±CI) with respect to the amount of projects $|P|$ and time window size γ

$\vert P\vert$	$\gamma=1$				$\gamma=2$				$\gamma=3$			
	ILS		**CPLEX**		**ILS**		**CPLEX**		**ILS**		**CPLEX**	
	sg'	$\pm CI$	*sg'*	$\pm CI$	*sg'*	$\pm CI$	*sg'*	$\pm CI$	*sg'*	$\pm CI$	*sg'*	$\pm CI$
10	2.31	0.54	2.04	0.51	3.90	0.41	3.14	0.30	4.87	0.47	3.87	0.54
20	**1.08**	0.10	0.78*	0.08	1.90	0.04	1.41	0.11	**2.74**	0.33	2.40*	0.17
30	**0.98**	0.14	0.61*	0.06	1.46	0.09	1.08	0.15	**1.58**	0.18	1.91*	1.06
40	**0.76**	0.15	0.65*	0.05	**1.02**	0.10	0.67*	0.22	**1.21**	0.13	2.71*	0.39
50	**0.67**	0.07	0.45*	0.03	**0.84**	0.11	0.87*	0.40	**1.04**	0.11	3.57*	0.47
70	**0.54**	0.07	0.33*	0.02	**0.64**	0.06	1.04*	0.16	**0.72**	0.11		
90	**0.58**	0.08	0.36*	0.14	**0.65**	0.06	1.00*	0.21	**0.61**	0.12		
110	**0.52**	0.06	0.35*	0.09	**0.58**	0.05	1.25*	0.24	**0.76**	0.13		
130	**0.48**	0.05	0.29*	0.10	**0.59**	0.05	1.22*	0.50	**0.62**	0.12		
150	**0.56**	0.07	0.34*	0.05	**0.49**	0.05	0.92*	0.22	**0.55**	0.04		
200	**0.45**	0.09	0.29*	0.10	**0.50**	0.07	0.95*	0.17	**0.49**	0.06		
250	**0.45**	0.13	0.43*	0.20	**0.55**	0.03			**0.92**	0.74		
300	**0.43**	0.05	0.44*		**0.47**	0.10			**0.51**	0.17		
Avg.	0.75	0.12	0.57	0.12	1.05	0.09	1.23	0.24	1.28	0.21	2.89	0.53

The results in Table 2 show that the average solution gap, over all of our test instances, for the heuristic and the MIP solver decreases with the amount of projects $|P|$. Applying the same utilization for all test instances causes the increase of projects to lead to a linear increase of work packages and a linear decrease of the demand per work packages. The higher amount of work packages and the lower demand per work package have positive smoothing effects on the capacity demand, which describes the latter solution gap decrease. The time window size γ has a positive correlation with the solution gap of both solution methods. Using the MIP solver as solution method, the presented results show that the percentage of unsolved instances $(1 - sa)$ increases with the amount of projects $|P|$ and the time window size γ.

The metaheuristic performs significantly better than the MIP solver. Specifically, the heuristic solution method solves all generated test instances with an average solution gap of 1.03%, whereas the exact solver returns an average solution gap of 1.39% for the approximately $\approx 70\%$ feasibly solved instances. The detailed values reflect the amount of projects $|P|$ and time window period γ. For a time window size of $\gamma = 1$ the MIP solver is slightly better (0.57%) than the developed heuristic (0.75%). Nevertheless, the solver fails to find a feasible solution for $\approx 55\%$ of instances. For a time window size of $\gamma = 2$ and $\gamma = 3$, the developed heuristic outperforms the solver in terms of both the solution gap *sg'* and solvability *sa*.

In Summary, with a predefined solution time, the *MIP-SP* model complexity is too high to use the MIP solver as an appropriate solution method. Even for small instances, the solver fails to return a feasible solution within a reasonable time. However, for small instances, the heuristic solution method performs as well as the exact solver does and also solves real-size instances, providing a good and stable solution gap.

5 CONCLUSION AND OUTLOOK

The MRCPSP according to (Kolisch and Heimerl, 2012) is extended, based on two practical model extensions. The performance of the exact solver is tested for the presented models for small to large and complex problems. One result is that the need for a heuristic solution method is illustrated due to the fact that the exact solver sometimes fails to return feasible integer solutions for the *MIP-SP* formulation.

The developed hybrid metaheuristic, for the *MIP-SP* model, provides for small and medium-sized problems favorable results comparing solution time, solution gap and solvability to the results of the exact solver. Additionally, for large problems, the solution gap, compared to the LP-relaxation of the model, stays steady while the exact solver fails to return feasible solutions.

In further research the hybrid metaheuristic solution method will be extended concerning additional stochastic influences.

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